MANDURAH CATHOLIC COLLEGE

Test 1 – Permutations and Combinations 2017

Section 1 Resource Free

MATHEMATICS SPECIALIST UNIT 1
Year 11

Name: _____

Teacher: _____

Result RF:_____/18 Result RR:_____/21

Total:_____/37 _____%

Time allowed for this section
Working time for this paper: 20 min + 25 min = 45 minutes

Materials required/recommended for this section
To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet

To be provided by the student

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters.

Special items: nil

Important note to students
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to students

1. **ALL** questions should be attempted.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you **do not use pencil**, except in diagrams.

Question 1 **(4 marks)**

(a) Evaluate $\frac{3!7!}{9!}$ (1 mark)

$= \frac{3!\,7!}{9 \times 8 \times 7!}$ ✓

$= \frac{6}{9 \times 8}$

$= \frac{2}{24} = \frac{1}{12}$ ✓

(b) Determine the values of a and b given $8! + 9! + 10! = a \times b!$ (3 marks)

$8! + 9! + 10!$

$= 8!(1 + 9 + 10 \times 9)$ ✓ factorises correctly

$\therefore a = 100$ ✓, $b = 8$ ✓

Question 2 **(5 marks)**

(a) Determine the number of different permutations of the word LETTER. (2 marks)

$= \frac{6!}{2!\,2!}$ ✓ uses repitition

$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 2}$

$= 180$ ✓

(b) A password is formed using all seven of the characters \$, %, @, Y, Z, 1 and 2 just once. Determine the number of different passwords that are possible in which all the symbols are adjacent, all the letters are adjacent and all the digits are adjacent. (3 marks)

$= 3! \times 3! \times 2! \times 2!$ ✓ groups correctly

$= 6 \times 6 \times 2 \times 2$

$= 144$ ✓ overall permutation

9

Question 3 **(3 marks)**

Show that if 50 different integers are selected from the set {1, 2, 3, ..., 97, 98}, there will be at least two integers whose sum is 99.

In the worst case if 49 numbers are chosen they would be the numbers 1 to 99. When the 50th number is chosen that 50th number must form a pair that adds to 99 with the numbers 1 to 49.

✓ uses pigeonhole principle

✓ correct labelling of pairs

✓ reasons correctly.

Question 4 **(6 marks)**

(a) Prove that ${}^{n}P_{r} = n \times {}^{n-1}P_{r-1}$. (3 marks)

$${}^{n}P_{r} = n \times \frac{(n-1)!}{(n-1-(r-1))!}$$ ✓ substitutes correctly

$$= \frac{n(n-1)!}{(n-1-r+1)!}$$ ✓ expands brackets correctly

$$= \frac{n!}{(n-r)!}$$ ✓ simplifies numerator correctly.

$$= {}^{n}P_{r} \quad \text{QED}$$

(b) If ${}^{9}P_{3} = 504$ and ${}^{10}P_{6} = 151200$, determine

(i) ${}^{9}P_{5}$ (1 mark)

$${}^{10}P_{6} = 10 \times {}^{9}P_{5}$$

$$151\,200 = 10 \times {}^{9}P_{5}$$

$$151\,20 = {}^{9}P_{5}$$ ✓

(ii) ${}^{11}P_{5}$ (2 marks)

$${}^{11}P_{5} = 11 \times {}^{10}P_{4}$$

$$= 11 \times 10 \times {}^{9}P_{3}$$ ✓ uses ${}^{9}P_{3}$

$$= 55\,440$$ ✓

END OF CALCULATOR FREE SECTION

MANDURAH CATHOLIC COLLEGE

Test 1 – Permutations and Combinations 2017

Section 2 Resource Rich

MATHEMATICS SPECIALIST UNIT 1
Year 11

Name: _____

Teacher: _____

Calculator Assumed:_____/21

Time allowed for this section

Working time for this paper: 20 min + 25 min = 45 minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the student

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters.

Special items: CAS calculator, scientific calculator, 1 A4 (1 sided) page of notes.

Important note to students

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to students

6. Write your answers in this Question/Answer Booklet.

7. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

8. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

9. It is recommended that you **do not use pencil**, except in diagrams.

Question 5 **(6 marks)**

A committee of 5 is to be chosen from 5 women and 7 men. How many committees are possible if:

(a) there are no restrictions? (1 mark)

$^{12}C_5 = 792$ ✓

(b) It must consist of 3 men and 2 women? (1 mark)

$^7C_3 \times {}^5C_2 = 350$ ✓

(c) There has to be more women than men. (2 marks)

3W, 2M: $^5C_3 \times {}^7C_2 = 210$

4W, 1M: $^5C_4 \times {}^7C_1 = 35$

5W, 0M: $^5C_5 \times {}^7C_0 = 1$ ✓ 3 situations.

Total $= 210 + 35 + 1$

$= 246$ ✓

(d) Two men refuse to be on the same committee? (2 marks)

Total $= 792 -$ (2 men together) $^2C_2 \times {}^{10}C_3$ ✓ uses complement

$= 672$ ✓

6

Question 6 **(4 marks)**

Determine the number of possible **four** letter permutations of the letters of the word

(a) RELOAD (1 mark)

$= \frac{6!}{2!}$

$= 360$ ✓

(b) RELOADED (3 marks)

All diff. letters = 360

2 Es, 2 diff. letters $= {}^{2}C_{2} \times {}^{5}C_{2} \times \frac{4!}{2!} = 120$

2 Ds, 2 diff. letters = 120 ✓ 2 diff., 2 same

2 Es, 2 Ds $= {}^{2}C_{2} \times {}^{2}C_{2} \times \frac{4!}{2!2!} = 6$ ✓ 2 pairs same

Total = 606 ✓

(4)

Question 7 **(4 marks)**

By considering the values in Pascal's triangle, or otherwise, find the value of a in each of the following:

(a) ${}^{n}C_{r} = \frac{n!}{(n-a)!r!}$ (1 mark)

$a = r$ ✓

(b) ${}^{12}C_4 = {}^{a}C_8$ (1 mark)

$a = 12$ ✓

(c) ${}^{10}C_6 + {}^{10}C_7 = {}^{11}C_a$ (1 mark)

$a = 7$ ✓

(d) $a = {}^{n}C_0 + {}^{n}C_n$ (1 mark)

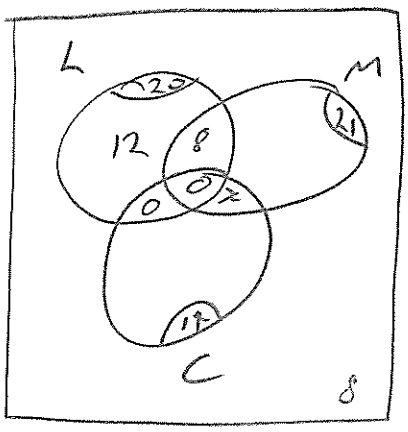
$a = 2$ ✓

4

Question 8 **(5 marks)**

In a survey of 45 Year 11 students, there were 20 students who studied Literature, 14 who studied Chemistry and 21 who studied Specialist Mathematics. No student studied both Literature and Chemistry. 8 studied none of these subjects. 8 students studied both Literature and Specialist Mathematics. Find the number of students who:

(a) Studied Specialist Mathematics only. (3 marks)



✓ Venn diagram correct

$37 = 20 + 21 + 14 - 8 - 0 - x + 0$

$x = 10$ ✓ uses inclusion-exclusion to find missing value

$\therefore$ only maths $= 3$ ✓

(b) Studies exactly one of these subjects. (2 marks)

Exactly one $= 12 + 4 + 3$ ✓ correctly calculates each subject

$= 19$ students ✓

5

Question 9 **(2 marks)**

Ten points are equally spaced around the circumference of a circle. Determine the number of simple (non-self-intersecting) convex polygons that can be formed by joining either three, four or five of these points with straight line segments.

Total = $\underbrace{{}^{10}C_3}_{3 \text{ points}} + \underbrace{{}^{10}C_4}_{4 \text{ points}} + \underbrace{{}^{10}C_5}_{5 \text{ points}}$ ✓ Addition

$= 582$ ✓

END OF ASSESSMENT

(2)